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The application of the real Weinberg state method to unbound levels in ^{13}C formed by neutron scattering and (d, p) on ^{12}C

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Abstract. The real Weinberg state method devised to calculate a suitable form factor of the resonant final state in a stripping reaction is applied to elastic resonant scattering of neutrons by ^{12}C . The purpose is to establish the methodological accuracy of the theory. Some important adjustments to the theory are incorporated. The phase shifts of the different total angular momentum and parity ($J^\pi = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$) channels are compared with the exact solution obtained from a coupled-channel method. The agreement is generally satisfactory. The parameters of the resonances yielded by the respective methods are also compared. The Weinberg method describes the levels as mixtures of discrete configurations which are either bsec (bound states embedded in the continuum) or single-particle resonances treated as Weinberg states. The wavefunctions of the respective methods are used as form factors in $^{12}\text{C}(\text{d}, \text{p})^{13}\text{C}$ stripping calculations, again with satisfactory agreement. The role of the Weinberg method as a schematic way of solving a coupled-channel problem is briefly discussed.

1. Introduction

It was found by Huby and Mines (1965) that the adaptation of the DWBA to handle stripping reactions $A + \text{d} \rightarrow \text{B}^* + \text{p}$ in which the residual nucleus B^* is left not in a bound state as usual, but in a quasi-stationary state or an unbound level above the particle emission threshold meant using a scattering wave for B^* in place of the usual bound one. This scattering wave should be an exact eigenfunction of the $(n + A)$ system in a scattering state. Recently, much theoretical as well as experimental effort has been directed towards the study of stripping to unbound states (Alty *et al* 1967, Bohne *et al* 1968, Huby and Liu 1968, Levin 1968, Vincent 1968, Bang and Zimanyi 1969, Bunakov 1970, Vincent and Fortune 1970, Lipperheide 1970, Fuchs *et al* 1971, 1972, Schlessinger and Payne 1972). Nearly all the authors used the DWBA formalism, but they differed in their prescriptions for the scattering wavefunction of the residual $(n + A)$ system.

The familiar problem of the slow convergence of the integral involving this scattering state in the DWBA was solved elegantly by Vincent and Fortune (1970). But up to the present, the scattering wavefunction of this B^* has been constructed in a rather *ad hoc* manner. One would like to see it being part of a more systematic theoretical scheme. Preferably, any such theory should endow this wavefunction with some features which are deemed desirable on very general grounds. These were previously described by Huby

(1971). We summarize here: (i) the wavefunction, hence scattering amplitude, should exhibit a Breit-Wigner type energy dependence, (ii) the radial function (the form factor) should have an almost bound behaviour inside the well, and make a smooth transition through the surface to the proper oscillatory form outside, (iii) a compound resonant level should exhibit a component which constitutes a single-particle (SP) potential resonance which is mixed with other configurations, (iv) it should embody a well defined spectroscopic factor.

These unbound states can also be reached by straightforward n - A resonant scattering. While there are many powerful, established resonant scattering theories, they are not particularly tailored to produce a simple and easily handled form factor for a resonant state which exhibits the above four features, is not too model dependent, and which experimentalists can incorporate into their DWBA programs so that stripping to bound and unbound states can be analysed in a unified way. In terms of accuracy and simplicity, this form factor should be comparable to the usual bound state ones calculated by the separation energy method (Austern 1970).

Let us consider how these requirements reflect on various theories and on our present choice of method. In the Humblet and Rosenfeld (1961) theory, the Gamow functions which are defined as the resonant wavefunctions have exponentially increasing asymptotic form: this applies also to the description of resonances by Glöckle *et al* (1967) and Romo (1968). The Feshbach theory falls short on the spectroscopic factor requirement because of the inexplicit way in which it mixes the SP resonance with the other configurations. A combination of the R matrix theory and Butler theory was successful in a very approximate analysis of stripping to unbound states (Huby and Liu 1968). In spite of recent development in R matrix theory (Purcell 1969), on its own the theory produces a form factor which is either too complicated for our purpose or is not smooth at the surface of separation of internal and external regions. The somewhat arbitrary device of a cut-off radius also occurs in the management of SP resonances as discrete states by Unger (1969), Wang and Shakin (1970), Mahaux and Saruis (1971).

Whereas the theories mentioned so far are to some extent 'schematic' in the desired sense of displaying resonance behaviour explicitly, though at the expense of accuracy, there is the opposite approach of setting up coupled-channel (CC) equations, given a sufficiently detailed model (eg within the continuum shell-model scheme), and solving these exactly, numerically across an energy range, to search for identifiable resonances in the results (eg Buck and Hill 1967, Mikoshiba *et al* 1971). This is laborious and gives little insight into the structure of the resonance, though recently procedures have been given for extracting spectroscopic information from CC calculations (Mikoshiba *et al* 1971, Mahaux and Saruis 1971), necessitating, however, the introduction of a cut-off radius. There are possibilities for simplifying CC calculations by matrix or finite rank approximations, such as those reviewed by Romo (1972), and the 'factorization' method of Van Giai and Marty (1970). However, the theories of this type so far seem to lack the particular degree of schematization needed to fulfil our stated requirements, but a way of achieving this was found by Huby (1971) through a suitable modification of the continuum shell model of Glöckle *et al* (1967) and Rosenfeld (1968). It must be emphasized that this is the motivation for introducing the modification into the theory of Glöckle *et al* rather than a wish to produce yet another continuum shell-model theory.

The actual important difference between Huby (1971) and Glöckle *et al* (1967) (and also Rosenfeld 1968) is the way in which the hamiltonian is manipulated. This leads to non-trivial modification of the formulation of the solution. The modified continuum shell model of Huby (1971) is developed in two different schemes which lead separately

to a T matrix and a K matrix. The latter scheme is more accurate and is the one which one of us adopted in a previous work (Liu 1973) and in the present paper. This scheme relies on the use of the real Weinberg state†.

Before a prescription for the form factor of the unbound state can be applied with confidence, the mathematical method used to derive it must be tested. Implicitly, this means testing the model of Huby (1971), in particular the crucial approximations introduced in the formalism and the real Weinberg state used to handle the sp resonance. In simple potential scattering, the accuracy of the real Weinberg state method was demonstrated by Liu (1973), from which further support was found for identifying a potential resonance by a real Weinberg state. The present paper deals with the situation where more complicated nuclear configurations are mixed in with the sp resonance. Assuming a suitable interaction model which describes $n-^{12}\text{C}$ scattering, our solution is compared with the exact solution, that is, that of a CC method. Our method is then regarded as an approximation to the CC method. If the latter falls short of predicting the actual experimental results, then so will ours. Quantities calculated in §§ 4 and 5 are the neutron phase shifts, the parameters of the ^{13}C levels, and also neutron wavefunctions, which are finally applied as form factors in a $DWBA$ calculation of the $^{12}\text{C}(d, p)^{13}\text{C}^*$ reaction leading to unbound levels. We examine in detail the extent to which the schematic approximation accounts for the physical behaviour of the unbound levels.

2. The real Weinberg state formalism

In this section we only describe the important changes in the formalism of Huby (1971) because of three major adjustments introduced in this work: (i) the decomposition of the space of functions, (ii) the construction of the separable potential from more than one real Weinberg state, (iii) the choice of energy at which the real Weinberg state is calculated. Some symbols have also been changed.

The underlying model of the $n-A$ scattering process is identical to that in Huby (1971). With trivial changes in symbols, the hamiltonian of the system is

$$H = H_A(\xi) + t + U_{sp}(\mathbf{r}) + V_{res}(\mathbf{r}, \xi), \quad (2.1)$$

whose eigenfunction is

$$\Phi(\mathbf{r}, \xi) = F_1(\mathbf{r})\Psi_{A1}(\xi) + \sum_{\sigma=2}^{\infty} F_{\sigma}(\mathbf{r})\Psi_{A\sigma}(\xi), \quad (2.2)$$

in which the first term is the elastic component and $\Psi_{A\sigma}$ is an eigenstate of $H_A(\xi)$.

It will prove convenient to use Weinberg treatment not only in the elastic channel as was done in Huby (1971). Therefore, besides the operator P which picks out the first term in (2.2) (and the operator $Q = 1 - P$), we also define a different projection operator \bar{P} which picks out several terms including the first, the exact channel selection being left open as yet. Then $\bar{Q} = 1 - \bar{P}$ projects out all the remaining terms. One can then follow the same operator and matrix equations as in Huby (1971) for finding the full standing-wave solution Φ and the K matrix, by a division of H into unperturbed and perturbed parts H_0 and H' , but doing this now with \bar{P} and \bar{Q} everywhere in place of P and Q . However, when it comes to the Weinberg method of splitting off a separable potential V_0 from the perturbation $\bar{P}H'\bar{P}$, we shall now construct a finite-rank potential

† The definition of the real Weinberg state is found in Liu (1973).

V_0 by using L in number quasi-particle states, which are to be specified later:

$$V_0 = \sum_{s,t}^L \bar{P}H'\bar{P}|s\rangle (A^{-1})_{st} \langle t|\bar{P}H'\bar{P}, \tag{2.3}$$

with

$$A_{st} = \langle \bar{s}|\bar{P}H'\bar{P}|t\rangle. \tag{2.4}$$

Under favourable conditions, $V' = \bar{P}H'\bar{P} - V_0$ can be a small quantity. This form of finite-rank potential was used by Fuller (1969) and Romo (1972), and differs from that of Glöckle *et al* (1967), Rosenfeld (1968), and Huby (1969), except when A_{st} is diagonal.

The result for G (the resolvent of the standing-wave Lippmann-Schwinger kernel) remains the same in form as (6.4) of Huby (1971), except that the summation over λ and μ runs from 1 to $(L+M)$, the states $|\alpha_\lambda\rangle$, $\langle \bar{\alpha}_\lambda|$ being enumerated as follows: the states

$$|\alpha_s\rangle = |s\rangle, \quad \langle \bar{\alpha}_s| = \langle \bar{s}|, \quad (s = 1, \dots, L) \tag{2.5}$$

are the quasi-particles, and the states

$$|\alpha_{L+i}\rangle = |Q_i\rangle, \quad \langle \bar{\alpha}_{L+i}| = \langle Q_i|, \quad (i = 1, \dots, M) \tag{2.6}$$

are \bar{Q} space eigenfunctions. Also because of (2.3), in the definition of the matrix element $N_{\lambda\mu}$ (equation (6.5) of Huby 1971), the term $A_\lambda\delta_{\lambda\mu}$ must be replaced by a suitably modified matrix element $A_{\lambda\mu}$.

The K matrix is†

$$K(E) = -\pi H'(1+G). \tag{2.7}$$

Next we specify that each quasi-particle state $|s\rangle$, $\langle \bar{s}|$ is to be a real Weinberg state (Weinberg 1963, Liu 1973) (a semi-ideal choice), $|\Phi_s(E)\rangle$, $\langle \Phi_s(E)|$ satisfying

$$\frac{P}{E-H_0} \bar{P}U_{sp}\bar{P}|\Phi_s(E)\rangle = \beta_s(E)|\Phi_s(E)\rangle \tag{2.8}$$

where Φ_s can be factorized,

$$\Phi_s(E, r, \xi) = \phi_s(E, r)\Psi_{A\sigma}(\xi). \tag{2.9}$$

Here P signifies principal part at the singularity of $(E-H_0)^{-1}$ for the open channel, but in a closed channel the regular behaviour of $(E-H_0)^{-1}$ must be used to obtain the correct asymptotic behaviour.

Then the matrix elements $N_{\lambda\mu}$ are: for $\lambda \leq L$, $\mu \leq L$, that is, when $|\alpha_\lambda\rangle$ and $|\alpha_\mu\rangle$ are in the \bar{P} space,

$$N_{\lambda\mu} \simeq (1-\beta_\lambda(E))\langle \alpha_\lambda|U_{sp}|\alpha_\mu\rangle\delta_{\lambda\mu} - (\beta_\lambda(E) + \beta_\mu(E) - 1)\langle \alpha_\lambda|V_{res}|\alpha_\mu\rangle \\ - \left\langle \alpha_\lambda \left| V_{res}\bar{P} \frac{\bar{P}}{E-H_0} \bar{P}V_{res} \right| \alpha_\mu \right\rangle; \tag{2.10a}$$

† This definition is identical to that of Newton (1966) but differs from that of Huby (1971) by a factor of $-\pi$.

for $\lambda > L, \mu > L$, that is, when $|\alpha_\lambda\rangle$ and $|\alpha_\mu\rangle$ are in the \bar{Q} space,

$$N_{\lambda\mu} \simeq (E - \tilde{E}_\lambda)\delta_{\lambda\mu} - \langle \alpha_\lambda | V_{\text{res}} | \alpha_\mu \rangle - \left\langle \alpha_\lambda \left| V_{\text{res}} \bar{P} \frac{P}{E - H_0} \bar{P} V_{\text{res}} \right| \alpha_\mu \right\rangle; \quad (2.10b)$$

for $\lambda \leq L, \mu > L$, that is, when $|\alpha_\lambda\rangle$ is in the \bar{P} space, $|\alpha_\mu\rangle$ is in the \bar{Q} space,

$$N_{\lambda\mu} = N_{\mu\lambda} \simeq -B_\lambda(E) \langle \alpha_\lambda | V_{\text{res}} | \alpha_\mu \rangle - \left\langle \alpha_\lambda \left| V_{\text{res}} \bar{P} \frac{P}{E - H_0} \bar{P} V_{\text{res}} \right| \alpha_\mu \right\rangle. \quad (2.10c)$$

Here we have relabelled the eigenvalues E_i of $\bar{Q}H_0\bar{Q}$ as \tilde{E}_λ (with $\lambda = L + i$). We made the same kind of approximation as in Huby (1971) to derive (2.10a–c), and with the corresponding approximations in (2.7) we obtain the on-shell matrix element of K in the free partial wave $|X\rangle$,

$$\tan \delta = -\pi \langle X | V' | X \rangle - \pi \sum_{\lambda, \mu=1}^{L+M} \langle X | H' | \alpha_\lambda \rangle (N^{-1})_{\lambda\mu} \langle \alpha_\mu | H' | X \rangle. \quad (2.11)$$

Use of (2.10)–(2.11) to calculate the phase shifts will be called method 1.

In these equations, there is implicit energy dependence in all the quantities, particularly in \mathbf{N} . In Huby (1971) the energy dependence near a resonance was exhibited approximately by making a linear expansion about the energy at which $\beta(E)$ is unity, that is, the formal single-particle resonance energy. However, more accurately the linear expansion may be made about any reference energy, say \bar{E} , near which we wish to work. The strongest energy dependence in $N_{\lambda\mu}$ will come from the factor $(1 - \beta_\lambda(E))$ in (2.10a). We therefore expand this linearly about \bar{E} , neglecting the energy dependence of all other factors in $N_{\lambda\mu}$. Provided $|\alpha_\lambda(\bar{E})\rangle$ is normalized so that

$$-\left. \frac{d\beta_\lambda(E)}{dE} \right|_{E=\bar{E}} \langle \alpha_\lambda(\bar{E}) | U_{\text{sp}} | \alpha_\lambda(\bar{E}) \rangle = 1, \quad (2.12)$$

we may then write

$$(1 - \beta_\lambda(E)) \langle \alpha_\lambda(E) | U_{\text{sp}} | \alpha_\lambda(E) \rangle \simeq E - \tilde{E}_\lambda, \quad (2.13)$$

where

$$\tilde{E}_\lambda = \bar{E} - (1 - \beta_\lambda(\bar{E})) \langle \alpha_\lambda(\bar{E}) | U_{\text{sp}} | \alpha_\lambda(\bar{E}) \rangle, \quad (\lambda = 1, \dots, L). \quad (2.14)$$

The energy \tilde{E}_λ is an 'effective' single-particle resonance energy, for use when E is near \bar{E} , and indeed \tilde{E}_λ will be almost equal to the formal single-particle resonance energy at which β_λ is unity, if $\beta_\lambda(E)$ varies nearly linearly with E . Now the matrix \mathbf{N} can be written in an approximate, compact form

$$\mathbf{N} = E\mathbf{1} - \mathbf{W} \quad (2.15)$$

where

$$W'_{\lambda\mu} = \tilde{E}_\lambda \delta_{\lambda\mu} + (\beta_\lambda(\bar{E}) + \beta_\mu(\bar{E}) - 1) \langle \alpha_\lambda(\bar{E}) | V_{\text{res}} | \alpha_\mu(\bar{E}) \rangle + \left\langle \alpha_\lambda(\bar{E}) \left| V_{\text{res}} \bar{P} \frac{P}{E - H_0} \bar{P} V_{\text{res}} \right| \alpha_\mu(\bar{E}) \right\rangle, \quad (2.16)$$

with the understanding that $\beta_\lambda(\bar{E}) = 1$ for any \bar{E} if $\lambda > L$. After \mathbf{W}' is diagonalized by an orthogonal matrix \mathbf{T} of elements $t_{\lambda\mu}$ its eigenvectors are

$$|\omega_\lambda\rangle = \sum_{\mu=1}^{L+M} t_{\mu\lambda} |\alpha_\mu\rangle \quad (2.17)$$

with eigenenergies $W'_\lambda(\bar{E})$. Each $|\omega_\lambda\rangle$ is associated with an unbound level of formal resonance energy $W'_\lambda(\bar{E})$ (if $W'_\lambda(\bar{E}) > 0$). After performing the transformation \mathbf{T} , (2.11) reduces to

$$\tan \delta = -\pi \langle X|V'|X\rangle - \sum_{\lambda=1}^{L+M} \frac{\Gamma'_\lambda(E)}{2(E - W'_\lambda(\bar{E}))}, \quad (2.18)$$

where

$$\begin{aligned} \Gamma'_\lambda(E) &= 2\pi \langle X(E)|H'|\omega_\lambda\rangle^2 \\ &\simeq \left(\frac{E}{W'_\lambda}\right)^{l+\frac{1}{2}} 2\pi \langle X(W'_\lambda)|H'|\omega_\lambda\rangle^2 \\ &= \left(\frac{E}{W'_\lambda}\right)^{l+\frac{1}{2}} \Gamma'_\lambda \end{aligned} \quad (2.19)$$

with Γ'_λ defined as the formal resonance width, the energy-dependence parametrization being taken from Liu (1973). Phase shifts calculated by (2.18), (2.19) are said to be calculated by method 2, that is, a fully schematic linear approximation to the more exact method 1.

If in the elastic scattering channel we have just one real Weinberg state Φ_1 , then a formal SP resonance width can be defined as

$$\Gamma'_{\text{sp}}(E) = 2\pi \langle X(E)|U_{\text{sp}}|\Phi_1(\bar{E})\rangle^2. \quad (2.20)$$

The spectroscopic amplitude of the λ th level may be defined as

$$\theta_\lambda = t_{1\lambda}. \quad (2.21)$$

The relation between the formal width of the λ th level and the SP formal width is, from (2.19)–(2.21),

$$\sqrt{\Gamma'_\lambda} = \theta_\lambda \sqrt{\Gamma'_{\text{sp}}} + \sqrt{2\pi} \langle X|V_{\text{res}}|\omega_\lambda\rangle, \quad (2.22)$$

(assuming that both the matrix elements appearing in (2.19) and (2.20) have been made positive by suitable phasing of $|\omega_\lambda\rangle$ and $|\Phi_1\rangle$). Under favourable circumstances this yields

$$\Gamma'_\lambda \simeq \theta_\lambda^2 \Gamma'_{\text{sp}}, \quad (2.23)$$

which constitutes a way of extracting the spectroscopic factor from the width.

Apart from minor but obvious changes in notation, we can proceed as in Huby (1971) to equations for an actual level width Γ_λ and shift Δ_λ and for the S matrix ((6.21)–(6.24) of Huby 1971).

The physical scattering wavefunction $\Psi^{(+)}$ which corresponds to the approximations of method 2 can be written as

$$\begin{aligned} \Psi^{(+)} &= \frac{1}{1 - i\langle X|K|X\rangle}(1 + G)X \\ &\simeq \pm e^{i\delta} \left\{ \frac{1}{(1 + \pi^2 \langle X|V'|X\rangle^2)^{1/2}} \left(1 + \frac{\mathbf{P}}{E - H_0} V' \right) |X\rangle \right. \\ &\quad + \sum_{\lambda=1}^{L+M} \left(\frac{\Gamma_\lambda}{2\pi} \right)^{1/2} \frac{1}{E - W'_\lambda} \left(\beta + \frac{\mathbf{P}}{E - H_0} \bar{P}V_{\text{res}} \right) |\omega_\lambda\rangle \left. \right\} \left\{ \left(1 - \sum_{\lambda=1}^{L+M} \frac{\Delta_\lambda}{E - W'_\lambda} \right)^2 \right. \\ &\quad \left. + \frac{1}{4} \left(\sum_{\lambda=1}^{L+M} \frac{\Gamma_\lambda}{E - W'_\lambda} \right)^2 \right\}^{-1/2}. \end{aligned} \quad (2.24)$$

Here β means an operator which, acting on any of the states $|\alpha_\lambda\rangle$, has the eigenvalue β_λ . (The (\pm) sign in (2.24) corresponds to the usual indeterminacy of phase shift by a multiple of π : it must be chosen to make $\pm \cos \delta$ positive.)

For widely-spaced resonances, a single-level approximation to (2.24) yields for the wavefunction in the elastic channel

$$\begin{aligned} P\Psi^{(+)} &\simeq \pm e^{i\delta} \left\{ \frac{E - W'_\lambda}{(1 + \pi^2 \langle X|V'|X\rangle^2)^{1/2}} \left(1 + \frac{\mathbf{P}}{E - H_0} PV' \right) |X\rangle \right. \\ &\quad \left. + \left(\frac{\Gamma_\lambda}{2\pi} \right)^{1/2} \left(\beta_1 \theta_\lambda \Phi_1(E) + \frac{\mathbf{P}}{E - H_0} PV_{\text{res}} |\omega_\lambda\rangle \right) \right\} \\ &\quad \times \{ (E - W'_\lambda - \Delta_\lambda)^2 + \frac{1}{4} \Gamma_\lambda^2 \}^{-1/2}. \end{aligned} \quad (2.25)$$

These wavefunctions satisfy the requirements set out in § 1, as in the similar case argued in Huby (1971), but now with extra refinements of the model listed at the beginning of this section. In particular $P\Psi^{(+)}$ has the right asymptotic behaviour. (A consistent convention for the (\pm) sign in (2.25) is to allow δ to vary continuously with energy from zero at zero energy, in which case the sign factor is $(-1)^\lambda$, on labelling the resonances λ as 1, 2, ... in ascending energy W'_λ .)

3. Application to n-¹²C scattering

The even-parity elastic scattering of neutrons on ¹²C was chosen as a test case for the real Weinberg state method. There is a wide energy range over which only elastic scattering occurs. In this range there are both wide and narrow resonances, indeed overlapping ones, which provide the situation we are looking for, that is, potential resonances mixed with BSEC. The corresponding spectrum of positive-parity levels in ¹³C is shown in figure 1, column 1. A collective model for ¹²C affords a reasonably successful description of n-¹²C scattering and is amenable both to CC calculations (Buck 1963, Pisent and Saruis 1967, Reynolds *et al* 1968, Pascolini *et al* 1969, Mikoshiba *et al* 1971) and to the present method. The most successful CC calculation on this reaction is by Mikoshiba *et al* (see also Mori and Terasawa 1972). To test the real Weinberg state method, assuming a given model, our calculations should be compared with the exact CC solution. To assume a model which gives a very good fit to the experimental data is not vitally important to us, and so we employed the simpler model of Pascolini *et al* (1969).

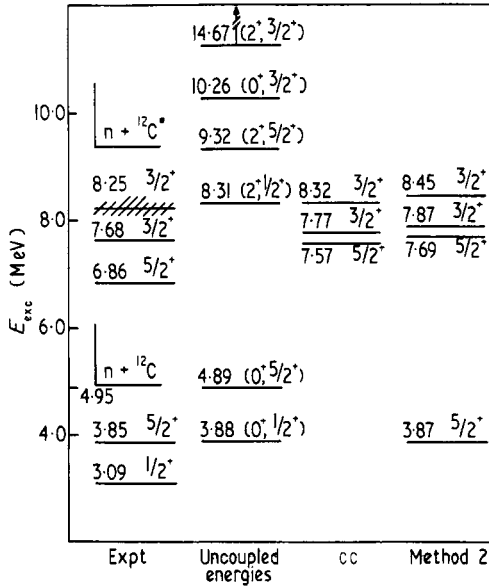


Figure 1. Positive-parity spectrum of ^{13}C in the energy region considered. The experimental data are from Aajenberg-Selove (1970).

The model describes ^{12}C as a rotator with two states, the 0^+ ground state and 2^+ excited state at 4.43 MeV, to the deformed field of which the incoming neutron is coupled. The neutron potential is of the usual deformed Woods-Saxon type plus spin-orbit term (Tamura 1965), and the parameters of Pascolini *et al* (1969) are used, in particular, deformation $\beta = -0.4$. A channel of total angular momentum (J, M_J) may be formed by vector addition of a rotational state of the core (angular momentum I, M_I and energy \mathcal{E}_I) to a neutron state with quantum numbers (j, l), the resulting radial wavefunction for the neutron in this partial-wave channel being written $u_{Ijl}^J(r)$. The wavefunctions for the various partial-wave channels with fixed J and parity are connected by a familiar set of coupled equations (Buck 1963, Pisent and Saruis 1967, Mikoshiba *et al* 1971, Tamura 1965):

$$\left\{ \frac{\hbar^2}{2m'} \left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} \right) - U_{sp} + E - \mathcal{E}_I \right\} u_{Ijl}^J(r) = \sum_{ijl} f_2(r) \omega_{I'j'l'; Ijl}^{\text{rot}} u_{I'j'l'}^J(r). \tag{3.1}$$

Here U_{sp} is the central plus spin-orbit component of the neutron potential, and $f_2(r)$ is the radial factor of the quadrupole-deformed component of the potential. The coupling matrix element ω^{rot} is a number given in Pisent and Saruis (1967, equation (17a)). The essence of the cc method is that these equations are integrated numerically with scattering boundary conditions, and from the asymptotic form of the solution the S matrix and the phase shift are deduced.

In the real Weinberg state method we consider an incident partial wave (j_1, l_1), and again we seek the 'exact' wavefunction which this induces in the various partial-wave channels having the same value of J (equals j_1) and parity. Equation (3.1) corresponds to an eigenstate equation for the hamiltonian of (2.1), the right hand side of (3.1) representing the effect of V_{res} . Neglecting V_{res} , we should obtain a set of uncoupled

bound states and resonances (I, j^π) consisting of a single-particle level j^π attached to a core state I .

The single-particle levels are found to be the following bound states—a $1s_{1/2}$ level at -23.81 MeV, a $2s_{1/2}$ at -1.07 MeV, and a $1d_{3/2}$ at -0.06 MeV—and a $d_{3/2}^3$ resonance at 5.31 MeV (the resonance being defined by the condition that the phase shift is $\frac{1}{2}\pi$). This would lead to the uncoupled bound states and resonances (I, j^π) in ^{13}C shown in figure 1, column 2 (assigning the experimental value 4.95 MeV to the neutron separation energy in ^{13}C).

For given J^π , the projection operator P selects the incident channel $(I, j^\pi) = (0, j_1^{\pi_1})$, but we wish further to set up a projection operator \bar{P} which in addition to these channels picks other partial-wave channels for Weinberg state treatment. The partial-wave channels with $d_{3/2}^3$ single-particle states call for this treatment because there is a $d_{3/2}^3$ resonance even though no bound state. The remaining channels form the \bar{Q} space, in which the basic states $|Q_i\rangle$ are the uncoupled bound states $|I, j^\pi\rangle$ already discussed, their angular momenta being summed vectorially to J . The resulting disposition of real Weinberg states $|\Phi_s\rangle$ and \bar{Q} space states $|Q_i\rangle$ is represented in figure 2, the former

| $J^\pi = 3/2^+$ | $J^\pi = 5/2^+$ | $J^\pi = 1/2^+$ |
|------------------------------------|------------------------------------|------------------------------------|
| 0 ⁺ ; 1d ^{3/2} | 0 ⁺ ; 1d ^{5/2} | 0 ⁺ ; 1s ^{1/2} |
| 2 ⁺ ; 1s ^{1/2} | 2 ⁺ ; 1s ^{1/2} | 0 ⁺ ; 2s ^{1/2} |
| 2 ⁺ ; 2s ^{1/2} | 2 ⁺ ; 2s ^{1/2} | 2 ⁺ ; 1d ^{3/2} |
| 2 ⁺ ; 1d ^{3/2} | 2 ⁺ ; 1d ^{3/2} | 2 ⁺ ; 1d ^{5/2} |
| 2 ⁺ ; 1d ^{5/2} | 2 ⁺ ; 1d ^{5/2} | |

Figure 2. The \bar{P} and \bar{Q} state vectors in the $J^\pi = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$ channels. The states in the shaded boxes are \bar{P} space state vectors, the others are in the \bar{Q} space.

being in the shaded boxes and the latter in the unshaded. The inclusion of the single-particle $1s_{1/2}$ state in this set looks dubious because of the Pauli principle, but it must be remembered that in the CC method all partial waves implicitly contain components of all principal quantum numbers. We can draw our conclusions after examining the calculation with and without this state.

The Weinberg states $|\Phi_s\rangle$ accord with (2.8) and (2.9) apart from summation of the angular momenta of the SP and core factors j and I to total J . When working at a reference energy \bar{E} , the SP neutron factor is a real Weinberg function as specified in Liu (1973, equation (2.3)) but now calculated at the SP energy

$$\bar{\mathcal{E}} = \bar{E} - \mathcal{E}_I. \tag{3.2}$$

We take the target ground state as energy datum, that is, $\mathcal{E}_0 = 0$. However, when $\bar{\mathcal{E}}$ is negative, that is, for the SP configuration $(2^+, 1d_{3/2}^3)$, we use the regular Lippman-

Schwinger kernel, (3.27) of Huby (1971), to define the Weinberg function. In any sp partial wave (j, l) at energy \mathcal{E} there is an infinite set of eigenfunctions ϕ_s of the above sp kernel. They may be labelled by a 'principal quantum number' $n = 1, 2, \dots$ in order of decreasing magnitude of the eigenvalue β . It is in this sense that a principal quantum number has been attached to the Weinberg states indicated in the shaded boxes of figure 2.

After inserting a r space representation of the principal part Green function in the formulae, all quantities are calculated numerically including the Weinberg function ϕ_s and its eigenvalue β_s .

In method 2, one will aim to choose a reference energy \bar{E} in the neighbourhood of a particular resonance W'_λ that is being studied, but if the method is to be meaningful the solution should be stable against variation of \bar{E} , that is, $W'_\lambda(\bar{E})$ should vary only weakly with \bar{E} .

A parametric formula must be used to extract resonance energies and widths from the phase shifts obtained from the experimental data, from the CC calculations and from method 1 so that these quantities can be meaningfully compared. We used a parametric formula suggested by (2.18)

$$\tan \delta = E^{l_1 + \frac{1}{2}} B + \sum_{\lambda=1}^{N_0} \frac{(E/W'_\lambda)^{l_1 + \frac{1}{2}} \Gamma'_\lambda}{2(W'_\lambda - E)}, \quad (3.3)$$

where N_0 is the number of resonances we are taking into account in the energy range considered. The quantities W'_λ are the energies where δ goes through odd multiples of $\frac{1}{2}\pi$. A least-squares fit determines the background coefficient B and the widths Γ'_λ .

4. Results: phase shifts and energy levels

All energies in this section are in the laboratory frame unless otherwise stated.

In figure 3, we plot β against the sp energy \mathcal{E} of the relevant partial waves. The curves have the same qualitative features as those in Liu (1973). Both the $1s\frac{1}{2}$ and $2s\frac{1}{2}$ solutions have β greater than unity at zero energy, which corresponds to the fact that the potential supports two bound s states, and this necessitates the use of both solutions as quasi-particles in the $J^\pi = \frac{1}{2}^+$ elastic-scattering channel. As shown by Liu (1973), these Weinberg states cannot be normalized by a real normalization factor.

For the $J^\pi = \frac{3}{2}^+$ channels, $\det[\mathbf{N}(E)]$ as a function of energy is plotted in figure 4, using (2.10a-c) for $\mathbf{N}(E)$. The exclusion of the configuration $(2^+, 1s\frac{1}{2})$ changes the shape of $\det[\mathbf{N}(E)]$ slightly, but not its zeros, which represent the resonance energies in our approximation, method 1. In figure 5, the $\frac{3}{2}^+$ phase shift from the CC method is compared with those from method 1 and method 2. The exclusion of the configuration $(2^+, 1s\frac{1}{2})$ in the latter two makes negligible difference to the results. The matrix \mathbf{W}' of (2.15) was calculated at $\bar{E} = 3.47$ MeV. For different choices of \bar{E} in the elastic region we obtain sets of eigenvalues W'_λ and the corresponding transformation coefficients $t_{\lambda\mu}$. For the states which are important to us, these quantities are rather independent of \bar{E} , as is desirable for the schematic approximation. The resonance energies and widths of the levels are presented in table 1 for comparison of experiment and the various theoretical methods. In method 2 the reference energy used was always $\bar{E} = 3.47$ MeV (except in dealing with level I of table 1(b) discussed below). The absolute values of the transformation coefficients $t_{\lambda\mu}$ for the $\frac{3}{2}^+$ levels I and II, as given by method 2, are included in table 2.

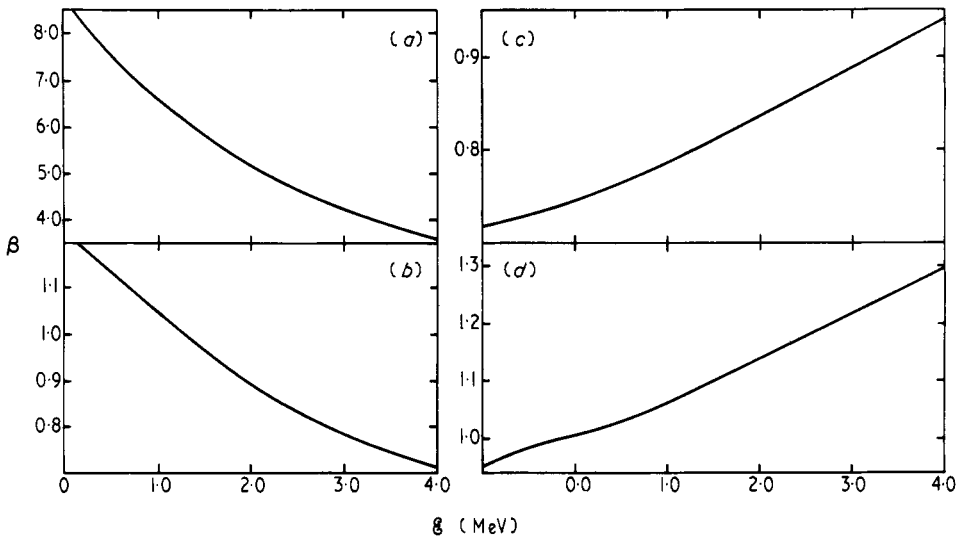


Figure 3. Plots of eigenvalues β of the real Weinberg states characterized by the neutron quantum numbers $nlj = (a) 1s_{1/2}, (b) 2s_{1/2}, (c) 1d_{3/2}$ and $(d) 1d_{5/2}$, as functions of sp energy ϵ .

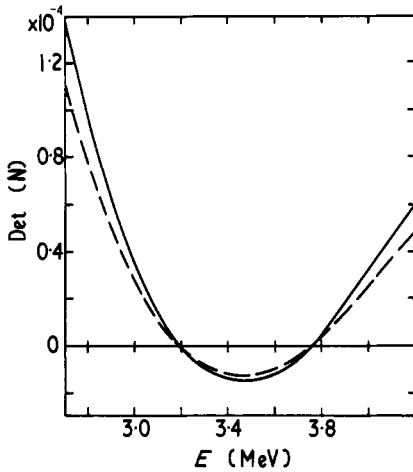


Figure 4. In the $J^\pi = \frac{3}{2}^+$ channel, plot of $\text{det}[N(E)]$ as a function of energy. Full curve $1s_{1/2}$ included, broken curve $1s_{1/2}$ not included.

These $\frac{3}{2}^+$ levels are the ones which appear in figure 1, columns 1, 3 and 4. They overlap substantially, and it can be seen that they are shifted quite far by V_{res} from the uncoupled levels of column 2. In particular, table 2 shows that the upper, broad level is a strong admixture of the $(0^+, 1d_{3/2}^3)$ sp resonance and the $(2^+, 1d_{5/2}^5)$ BSEC whose uncoupled energies lie well removed from it. The results of the real Weinberg state methods 1 and 2 resemble each other closely. They yield phase shift curves in figure 5 which run a course similar to that of CC, and correspondingly the parameters of the $\frac{3}{2}^+$ levels I and II in table 1(a) are generally similar as given by the CC and Weinberg calculations, though there is indeed some displacement between the CC and Weinberg resonance energies, and a difference between the widths. The last two rows of table 1(a) show that the crude sp

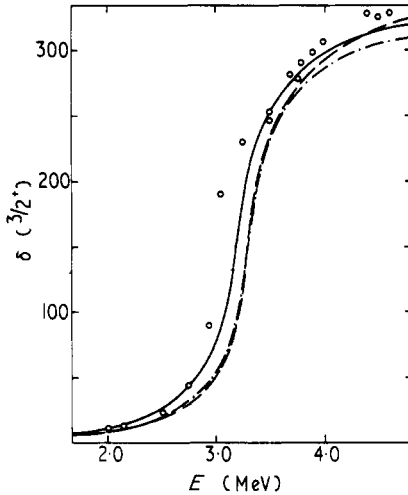


Figure 5. In the $J^\pi = \frac{3}{2}^+$ channel, plots of the phase shift as function of energy. The small circles are from experiments (Pascolini *et al* 1969), the full curve is from the CC method, the broken curve is from method 1, the chain curve is from method 2. The latter two plots remain unchanged whether $(2^+, 1s\frac{1}{2})$ is included or not.

approximation $\theta_\lambda^2 \Gamma'_{sp}$ to the level width Γ'_λ (2.22) is not very good, perhaps suggesting that the smaller the spectroscopic factor θ_λ^2 the worse this approximation.

The $J^\pi = \frac{5}{2}^+$ channels are interesting in that the potential in the elastic channel $(0^+, 1d\frac{5}{2})$ supports a bound state, though we describe it instead by a real Weinberg state. The zero of $\det[\mathbf{N}(E)]$ below the inelastic threshold depends hardly at all on whether or not the $(2^+, 1s\frac{1}{2})$ configuration is included. The various results for the $\frac{5}{2}^+$ phase shift are compared in figure 6. Comments on the eigenvalues W'_λ and $t_{\lambda\mu}$ as functions of \bar{E} are the same as in the $\frac{3}{2}^+$ channel. The absolute values of $t_{\lambda\mu}$ for the $\frac{5}{2}^+$ levels I and II, as given by method 2 neglecting the configuration $(2^+, 1s\frac{1}{2})$, are included in table 2. Level I corresponds to the bound $\frac{5}{2}^+$ state shown in figure 1, columns 1 and 4. To calculate it accurately we ought to work at a negative reference energy \bar{E} making the appropriate analytic continuation, but in fact the calculation was done at $\bar{E} = 0.2$ MeV. Also the Pauli principle whose effect may be greater for bound states, ought to be included, but this is beyond the scope of the model. We only note here that the level is not entirely single particle in nature. Level II represents the narrow $\frac{5}{2}^+$ resonance appearing in figure 1, columns 1, 3 and 4. It is mainly a mixture of two BSEC's, with a small admixture of the open-channel level $(0^+, 1d\frac{5}{2})$, its energy being well removed from all these uncoupled levels. As in the $\frac{3}{2}^+$ case, the results of methods 1 and 2 agree closely with each other, and they agree in their general features with the CC results, as expressed by the phase-shift curves in figure 6 and the parameters of level II in table 1(b), but there is some displacement between the resonance energies given by the respective methods and a difference between their level widths. Other remarks made about the $\frac{3}{2}^+$ results also apply to the $\frac{5}{2}^+$ case.

It must be recognized that, while the various theoretical results for the $\frac{5}{2}^+$ channels agree quite well with each other, they are all way off the experimental data. For the $J^\pi = \frac{1}{2}^+$ channel, the exclusion of the $(0^+, 1s\frac{1}{2})$ configuration changes significantly the shape of $\det[\mathbf{N}(E)]$ as a function of E . This difference is demonstrated dramatically in the phase shift plot of figure 7. It also shows that in this channel the inclusion of this

Table 1. Comparison of the resonance energies, widths etc, obtained from experiment, the CC method and methods 1 and 2 for (a) $\frac{3}{2}^+$ channels and (b) $\frac{5}{2}^+$ channels

| (a) $J^\pi = \frac{3}{2}^+$ channels | | | | | |
|---|---|---|---|------------------------|-------|
| Level number | I | II | III | IV | |
| Dominant configurations (I, nlj) | $\left\{ \begin{array}{l} (2^+, 2s\frac{1}{2}) \\ (2^+, 1d\frac{3}{2}) \end{array} \right.$ | $\left\{ \begin{array}{l} (0^+, 1d\frac{3}{2}) \\ (2^+, 1d\frac{3}{2}) \end{array} \right.$ | $\left\{ \begin{array}{l} (0^+, 1d\frac{3}{2}) \\ (2^+, 1d\frac{3}{2}) \end{array} \right.$ | $(2^+, 1d\frac{3}{2})$ | |
| Resonance energy W'_λ (lab, MeV) | Experiment | 2.95 | 3.58 | — | — |
| | CC | 3.06 | 3.65 | — | — |
| | Method 1 | 3.19 | 3.76 | — | — |
| | Method 2 | 3.17 | 3.79 | 6.83 | 15.79 |
| Width Γ'_λ (keV) | Experiment | 250 | 960 | — | — |
| | CC | 410 | 1100 | — | — |
| | Method 1 | 310 | 1090 | — | — |
| | Method 2 | 350 | 1210 | — | — |
| $\theta_\lambda^2 \Gamma'_{sp}$ (keV) | 230 | 1000 | — | — | |
| θ_λ | 0.36 | 0.63 | 0.67 | -0.18 | |
| (b) $J^\pi = \frac{5}{2}^+$ channels | | | | | |
| Level number | I† | II | III | IV | |
| Dominant configuration (I, nlj) | $\left\{ \begin{array}{l} (0^+, 1d\frac{3}{2}) \\ (2^+, 2s\frac{1}{2}) \end{array} \right.$ | $\left\{ \begin{array}{l} (2^+, 2s\frac{1}{2}) \\ (2^+, 1d\frac{3}{2}) \end{array} \right.$ | $(2^+, 1d\frac{3}{2})$ | $(2^+, 1d\frac{3}{2})$ | |
| Resonance energy W'_λ (lab, MeV) | Experiment | -1.09 | 2.08 | — | — |
| | CC | — | 2.84 | — | — |
| | Method 1 | — | 2.96 | — | — |
| | Method 2 | -1.08 | 2.97 | 5.19 | 15.25 |
| Width Γ'_λ (keV) | Experiment | — | 11 | — | — |
| | CC | — | 17 | — | — |
| | Method 1 | — | 14 | — | — |
| | Method 2 | — | 12 | — | — |
| $\theta_\lambda^2 \Gamma'_{sp}$ (keV) | — | 9 | — | — | |
| θ_λ | 0.92 | 0.08 | 0.36 | 0.09 | |

† The energies for this bound state are CM energies.

configuration is essential for achieving correct results. This is a peculiarity of the Weinberg state formalism which perhaps detracts from its intuitive simplicity and appeal. The case of s wave neutron scattering does not lend itself to the schematic treatment of method 2.

In summary, the real Weinberg state approximation reproduces quite faithfully the phase-shift curves and the resonance parameters as given by the exact CC calculation, though the accuracy of the agreement is not always very high. The test is an exacting one, because in this model V_{res} produces large energy shifts and admixtures of the uncoupled states, together with overlapping levels. The results of the more exact but less illuminating method 1 are approached closely by those of the schematic approximation, method 2.

Table 2 presents a comparison of the absolute values of the mixing coefficients $|t_{\lambda\mu}|$ of method 2 for four levels with the corresponding quantities from a shell-model calculation (Sebe 1963) and a CC method (Mikoshiba *et al* 1971). There is a considerable measure of agreement in the structural description of the levels by the different theories,

Table 2. Spectroscopy of the four levels considered. The absolute values of the transformation matrix elements $t_{\lambda\mu}$ from method 2 are compared with the absolute values of the mixing coefficients from a shell-model calculation (Sebe 1963), and with the absolute values of the square roots of the 'mixing ratio' from a cc calculation of Mikoshiba *et al* (1971)

| Excitation energy from method 2 (MeV) | J^π | | $(2^+, 1d_{3/2})$ | $(2^+, 1d_{5/2})$ | $(2^+, 2s_{1/2})$ | $(0^+, 1d_{3/2})$ | $(0^+, 1d_{5/2})$ |
|---------------------------------------|-----------------|-------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| 8.45 | $\frac{3}{2}^+$ | Method 2 | 0.714 | 0.186 | 0.243 | 0.630 | |
| | | Shell model | 0.608 | 0.366 | 0.154 | 0.685 | |
| | | cc | 0.597 | 0.346 | 0.032 | 0.722 | |
| 7.87 | $\frac{3}{2}^+$ | Method 2 | 0.004 | 0.003 | 0.931 | 0.364 | |
| | | Shell model | 0.062 | 0.013 | 0.955 | 0.282 | |
| | | cc | 0.148 | 0.21 | 0.929 | 0.270 | |
| 7.69 | $\frac{5}{2}^+$ | Method 2 | 0.545 | 0.011 | 0.835 | | 0.079 |
| | | Shell model | 0.465 | 0.002 | 0.873 | | 0.118 |
| | | cc | 0.531 | 0.071 | 0.843 | | 0.055 |
| 3.87 | $\frac{5}{2}^+$ | Method 2 | 0.360 | 0.045 | 0.126 | | 0.920 |
| | | Shell model | 0.402 | 0.063 | 0.095 | | 0.908 |
| | | cc | 0.437 | 0.105 | 0.130 | | 0.835 |

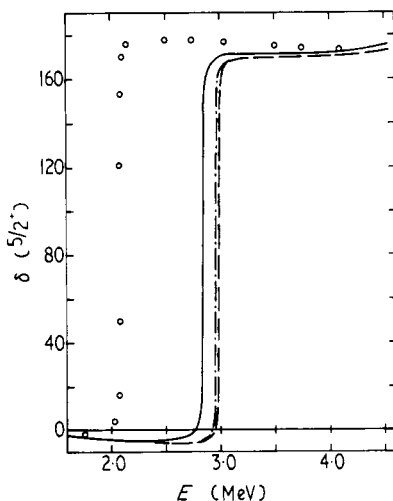


Figure 6. In the $J^\pi = \frac{5}{2}^+$ channel, plots of the phase shift as function of energy. Small circles are from experiments (Pascolini *et al* 1969), the full curve is from cc, the broken curve is from method 1, the chain curve is from method 2. The latter two plots remain unchanged whether $(2^+, 1s_{1/2})$ is included or not.

though the shell model neglects the unbound nature of the levels, and the cc calculation (Mikoshiba *et al* 1971) necessitated the artifice of a radial cut-off of the wavefunctions, in contrast to the consistent, schematic description of method 2.

A really simple interpretation of the resonances would be realized if the width formula (2.22) were well approximated by (2.23), so that the width Γ_λ was just a fragment

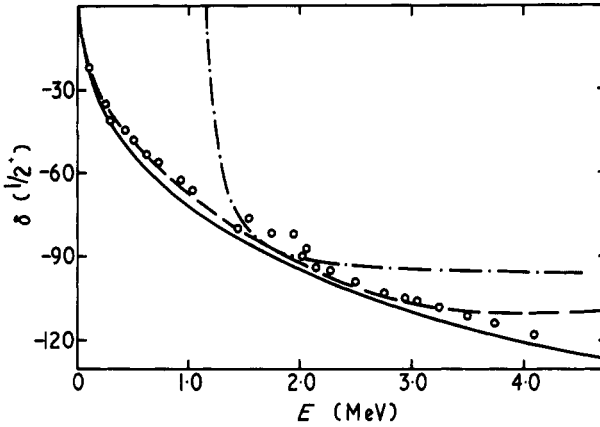


Figure 7. In the $J^\pi = \frac{1}{2}^+$ channel, plots of the phase shift as function of energy. The small circles are from experiments (Pascolini *et al* 1969), the full curve is from CC, the broken curve is from method 1 with $(0^+, 1s\frac{1}{2})$ included, the chain curve is from method 1 with $(0^+, 1s\frac{1}{2})$ excluded.

of the SP width with spectroscopic factor θ_λ^2 . The first term in (2.22) is relatively 'model independent' by contrast with the second term, a 'model-dependent' correction due to V_{res} . However, our results show that the 'model-dependent' term is in fact important, and so the simplest interpretation is dubious.

The present calculations may be compared with some of the many others on $n-^{12}\text{C}$ scattering amplitudes in the same energy range, some of which have indeed achieved closer agreement with experiment. On solving the CC equations of a similar but more sophisticated model, Mikoshiba *et al* (1971) were able to get the first $\frac{5}{2}^+$ resonance at the right energy by including a specially prescribed interaction. We are confident that, by applying our approximations, methods 1 and 2, to their model we could have essentially reproduced their results.

The Feshbach theory has been applied with varying degrees of success. Lovas (1966) obtained quite good agreement with experiment, but his results have been brought into question (Reynolds *et al* 1968, Robson and Van Megen 1972a). Robson and Van Megen (1972a, b) seemed unable to make their resonance levels tally recognizably with a structure-model calculation which represented all the levels as discrete, in the way that table 2 succeeds in doing. Leung and Koshel (1973) obtained good agreement with experiment, using a rather more involved Nilsson-type wavefunction.

R matrix calculations have been taken far enough to offer promise of success if elaborated sufficiently (Buttle 1967, Purcell 1969, Robson and Van Megen 1972a, b).

The application of our method to this reaction seems to show up as a favourable compromise between simplicity and accuracy, apart from illustrating the general possibilities of the method.

5. Wavefunction and stripping cross sections

The unbound states of ^{13}C have been seen as final states in the $^{12}\text{C}(d, p)^{13}\text{C}$ reaction in two experiments (McGruer *et al* 1955, Hosono 1968). Hosono saw a $\frac{5}{2}^+$ level at 6.86 MeV and $\frac{3}{2}^+$ levels at 7.64 MeV and 8.33 MeV; McGruer *et al* measured the same levels at excitation energies 6.87, 7.64 and 8.4 MeV respectively. Unfortunately, there

is some doubt about the reliability of the experimental results; firstly the cross sections quoted for the $\frac{5}{2}^+$ level differ by a factor of almost two in the two experiments, despite almost identical conditions. Secondly, the $\frac{3}{2}^+$ levels may not have been analysed properly: they are wide overlapping levels which interfere, and hence energy integrated cross sections for the individual levels cannot be extracted in the usual way. For these reasons, all calculations in this section are compared only with the corresponding coupled-channel results, not with experiment.

Wavefunctions in the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ channels have been calculated with the aid of (2.24). It is $u_{0j_{11}}^f(r)$, the radial part† of the open-channel component $P\Psi^{(+)}$, which must be used in stripping calculations. This contains four terms: one u_{R1} , proportional to the sp Weinberg state (3rd term in curly bracket), a correction u_{R2} to this resonant term (last term), depending on the coupling V_{res} , a main background term u_{B1} (1st term) proportional to the free wave, and a correction u_{B2} (2nd term) depending on V' . The component of $\Psi^{(+)}$ in the subspace spanned by the configuration $(2^+, \frac{3}{2}^+)$, which is treated as a quasi-particle, has no term u_{B1} , and u_{B2} is very small, depending only on V_{res} , not U_{sp} . In the subspace spanned by $|Q_i\rangle$, only the main term u_{R1} exists, proportional to the $|Q_i\rangle$.

All wavefunctions were calculated for a range of neutron energies E , putting $\bar{E} = E$ and recalculating everything (including $\Gamma_\lambda(E)$) at each energy, so that, in effect, no linear approximation was made. Open-channel wavefunctions are normalized asymptotically,

$$u(r) \sim \sin(kr - \frac{1}{2}l_1\pi + \delta). \quad (5.1)$$

It should be remembered that resonance energies from the Weinberg state method and CC method differ by over 100 keV (table 1), therefore wavefunctions compared in this section are calculated at or near the resonance energies of the appropriate method—comparison at other energies would be misleading.

Stripping cross sections and angular distributions were also calculated as functions of E , for a hypothetical (d, p) reaction on ^{12}C . The deuteron energy was kept constant at 14.6 MeV (lab) and the energy of the neutron (and hence also of the outgoing proton) was varied in a region about each of the resonances. The wavefunctions $u(r)$ and their coupled-channel counterparts were substituted in the DWBA matrix elements, the convergence of the radial integrals being expedited by the method of contour integration due to Vincent and Fortune (1970). Figure 8 shows how the wavefunction $u(r)$ depends on each of the constituent parts in the $\frac{5}{2}^+$ channel at an energy only 0.6 keV above level II (W'_λ as calculated by the Weinberg approximation, table 1(b), method 2). The background u_{B2} is too small to show: away from the level u_{B1} becomes larger and eventually dominates. Whereas in the $\frac{5}{2}^+$ channel the shape of u_{R2} inside about 5 fm changes strongly with energy, producing a resulting change in the total wavefunction (particularly near the origin), in the $\frac{3}{2}^+$ channel the effect is not so marked. The relative magnitude of the correction terms u_{R2} and u_{B2} to u_{R1} and u_{B1} respectively remains approximately constant at about 10–15%. Figures 9(a), (b) compare the open-channel wavefunctions with the corresponding CC calculation near level I in the $\frac{3}{2}^+$ channel (table 1(a)) and level II in the $\frac{5}{2}^+$ channel (table 1(b)). In the $\frac{5}{2}^+$ case the discrepancy at small radii is due to the effect noted above—it becomes larger as the energy decreases. In the $\frac{3}{2}^+$ channel near level II the two calculations agree to better than 2% in the range 1–9 fm. The closed-channel wavefunctions have also been compared. For the component in the space spanned by the configuration $(2^+, \frac{3}{2}^+)$ the agreement is similar to that for $P\Psi^{(+)}$. For the other components agreement is usually not so good, which reflects the

† Abbreviated subsequently to $u(r)$.

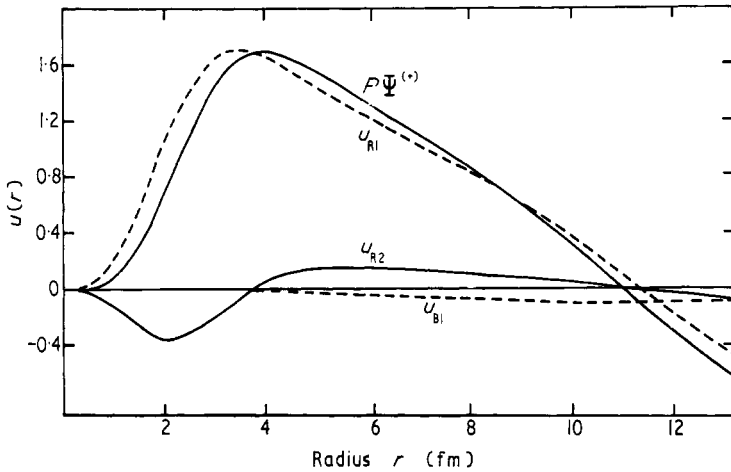


Figure 8. In the $J^\pi = \frac{5}{2}^+$ channel, plots of the constituent parts of the open-channel radial wavefunction at $E = 2.737$ MeV(CM) together with their sum $P\Psi^{(+)}$.

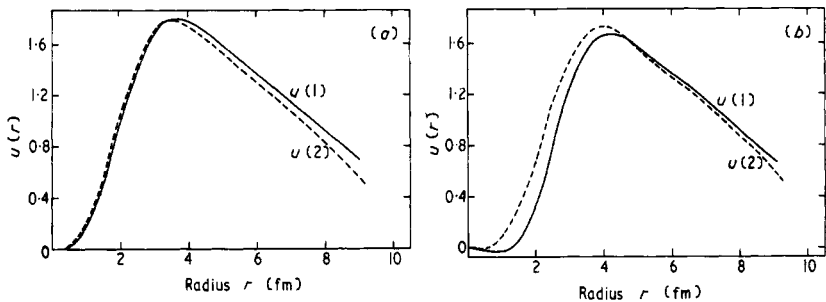


Figure 9. (a) In the $J^\pi = \frac{3}{2}^+$ channel, plots of the open-channel radial wavefunctions from the CC calculation, $u(1)$, and Weinberg calculation, $u(2)$, at CM energies 2.815 MeV and 2.95 MeV respectively. The corresponding energies for the level I (table 1(a)) are 2.817 MeV and 2.943 MeV. (b) as for (a) but in the $J^\pi = \frac{5}{2}^+$ channel at CM energies 2.6232 MeV and 2.737 MeV respectively. The corresponding energies for level II (table 1(b)) are 2.6227 MeV and 2.7364 MeV.

fact that their shape is uninfluenced by the interaction V_{res} . In general, agreement is better for the $\frac{3}{2}^+$ levels than for the $\frac{5}{2}^+$ level. Stripping cross sections for the $^{12}\text{C}(d, p)$ reaction, calculated at 0° , are compared in figures 10(a), (b) for the $\frac{3}{2}^+$ and $\frac{5}{2}^+$ channels respectively. The $\frac{3}{2}^+$ shows how the two levels interfere strongly near $E = 3.1$ MeV (CM). The Weinberg state method reproduces rather well the coupled-channel results except for the shift in energy of about 0.1 MeV. This shift is even more striking in the $\frac{5}{2}^+$ channel since it is much larger than the level width. The greater width from the coupled-channel calculations (table 1(b)) is also evident.

For the angular distribution, the two calculations agree to within about 5% (they also agree quite well with experiment (Hosono 1968)). Also, the shape does not vary too much with energy within the width of a level, thus making it easier to integrate over neutron energy.

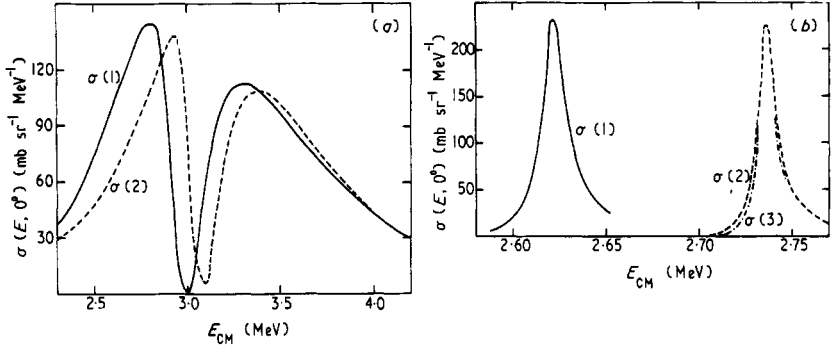


Figure 10. In the $J^\pi = (a) \frac{3}{2}^+, (b) \frac{5}{2}^+$ channels, plots of the zero-angle DWBA cross sections for the reaction $^{12}\text{C}(d, p)^{13}\text{C}$ as functions of neutron energy. $\sigma(1)$ and $\sigma(2)$ result from cc wavefunctions and (2.24) respectively. In (b) $\sigma(3)$ is the model-independent calculation described in § 5.

One would hope that for an isolated resonance the energy dependence of the cross section $\sigma(E)$ should be given approximately by

$$\sigma(E, \theta) = \frac{\frac{1}{4}\Gamma_\lambda^2(W'_\lambda)}{(E - \Delta_\lambda - W'_\lambda)^2 + \frac{1}{4}\Gamma_\lambda^2(W'_\lambda)} \sigma(W'_\lambda, \theta) \quad (5.2)$$

(cf equation (2.25) for the corresponding wavefunction). The energy-integrated cross section is then given by

$$\frac{1}{2}\pi\Gamma_\lambda(W'_\lambda)\sigma(W'_\lambda, \theta). \quad (5.3)$$

These formulae have been checked numerically for the $\frac{5}{2}^+$ level II, where, although the energy dependence is not given very accurately by (5.2) (shown by the asymmetry in figure 10(b)), the integrated values are within 3% of those predicted by (5.3). This holds for both the coupled-channel and Weinberg results, although these differ since the corresponding widths differ (table 1(b)). This calculation is not possible for the $\frac{3}{2}^+$ channel where the levels interfere destructively.

In the $\frac{5}{2}^+$ channel, the energy-integrated stripping cross sections calculated with the many-level equation (2.24) and a single-level approximation to it differ only by 3%, and the former is within 1% of the value calculated from (5.2). Therefore, in spite of the noticeable effect of the -1.08 MeV bound state on the wavefunction, a single-level approximation of the wavefunction is regarded as accurate enough for the extraction of the spectroscopic factor. Although the background terms may affect the wavefunction considerably, it has been estimated that they contribute only approximately 5–10% of the energy-integrated cross section.

Another common approximation is the neglect of the small correction terms u_{B2} and u_{R2} in the wavefunction, since the calculation of these requires a detailed knowledge of the nuclear structure ('model dependence'). Calculations show that for this reaction these can indeed be neglected provided the resulting wavefunction is renormalized to the asymptotic magnitude given by (5.1). The calculated cross sections are then surprisingly accurate although the approximate wavefunction may be wrong for small radii, particularly for the $\frac{5}{2}^+$ level (it is well known that DWBA calculations are insensitive to such radii).

For the $\frac{5}{2}^+$ channel this approximate cross section is compared in figure 10(b) with the cross section when all terms are retained. The accuracy of the approximation in the $\frac{3}{2}^+$ channel is similar, showing that even when there are overlapping resonances the analysis can still be performed in this model-independent way.

6. Conclusion

The real Weinberg state calculations succeeded in reproducing the features of the CC calculations for both the n - ^{12}C elastic scattering phase shift and level parameters in § 4, and the wavefunctions required as form factors for $^{12}\text{C}(d, p)^{13}\text{C}$ stripping in § 5, though with varying degrees of accuracy. The schematic method 2 was almost as good as the more exact method 1. The test was a stringent one because of the complexity of the spectrum, with strong configuration mixing. The fundamental requirement of a sufficiently simple and proper form factor, imposed in § 1, is met in (2.24) and (2.25), in which the third and fourth terms usually dominate. Under sufficiently favourable conditions, the fourth 'model-dependent' term might be neglected, leading to very simple extraction of the spectroscopic factor. However, we found that in the example calculated this was not good enough. Nevertheless, the last calculation in § 5 indicates a practical means of overcoming this, which it is intended to pursue elsewhere. These corrections correspond to the nuclear structure effects which have been much studied recently in calculating form factors for stripping to bound states by the separation energy prescription. In this connection, the work of Anderson *et al* (1970) runs parallel to ours in their introduction of Sturm-Liouville states (eigenstates of the Lippmann-Schwinger kernel in another guise) to calculate bound-state form factors.

From the viewpoint of mathematical technique, while our more exact method 1 offers less insight into the process than the schematic method 2, it can be regarded as an algebraical approximation for solving a CC problem by means of inverting a small matrix, in the category of methods studied by Romo (1972). The method of real Weinberg states leaves room for various developments. Thus it is possible to treat *all* channels by Weinberg methods, that is, to absorb the Q space into the P space, which is perhaps a more consistent approach, coming closer to the work of Anderson *et al* (1970). Calculations of this kind have been performed, but the numerical results were only slightly affected.

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